

# Time-varying Pedestrian Flow Models for Service Robots

Tomáš Votr<sup>1</sup>, Sergi Molina<sup>2</sup>, Ransalu Senanayake<sup>3</sup>, George Broughton<sup>1</sup>, Zhi Yan<sup>4</sup>, Jiří Ulrich<sup>1</sup>,  
Tomasz Piotr Kucner<sup>5</sup>, Chittaranjan Srinivas Swaminathan<sup>5</sup>, Filip Majer<sup>1</sup>, Mária Stachová<sup>6</sup>,  
Achim J. Lilienthal<sup>5</sup> and Tomáš Krajník<sup>1</sup>

**Abstract**— We present a human-centric spatiotemporal model for service robots operating in densely populated environments for long time periods. The method integrates observations of pedestrians performed by a mobile robot at different locations and times into a memory efficient model, that represents the spatial layout of natural pedestrian flows and how they change over time. To represent temporal variations of the observed flows, our method does not model the time in a linear fashion, but by several dimensions wrapped into themselves. This representation of time can capture long-term (i.e. days to weeks) periodic patterns of peoples’ routines and habits. Knowledge of these patterns allows making long-term predictions of future human presence and walking directions, which can support mobile robot navigation in human-populated environments. Using datasets gathered for several weeks, we compare the model to state-of-the-art methods for pedestrian flow modelling.

## I. INTRODUCTION

The advances in artificial intelligence, machine vision, and computer science, along with the ever-decreasing prices of hardware allowed the introduction of robots into domestic and office environments. These robots are supposed to share their space with people, interact with them, and perform tasks which are considered to be monotonous, tedious, or boring. However, introducing mobile robots into human environments faces several challenges.

One of such challenges is the reliability and safety of long-term operation in environments that change over time due to people activity. Unless properly addressed, the environment changes cause gradual deterioration of robot localisation robustness and, in turn, navigation efficiency. The effect of changes can be suppressed by gradual adaptation of the

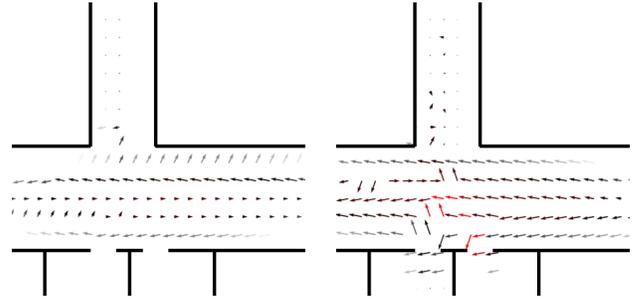


Fig. 1. Directions of pedestrian movement at 9:15 and 16:30 predicted by the proposed model. The arrow lengths correspond to flow intensity, i.e. number of people walking in the directions indicated by the arrows.

spatial environment models [1]–[4] or by explicit representation of time, which allows to model persistence [5], [6], periodicity [7] or more general long-term dynamics [8].

Another challenge is acceptance of the robots by the humans, who might consider the robots to behave in an inappropriate, offensive, or even aggressive way. As pointed out in [9], one of the critical aspects of long-term acceptance of mobile robots in human-populated environments was the way they navigate around humans. One of the problems is that nowadays, navigation methods represent the environment as a static structure and dynamic objects, such as humans, are treated separately. That assumes a reactive approach, where a robot estimates the people velocities by tracking them and then replans its trajectory. As reported in [10], the errors of state-of-the-art methods exceed 0.4 m for prediction horizons of 1 s, which means that reactive navigation around people still requires a high-speed sense-predict-plan-act loop.

To overcome the limitations of reactive approaches, a robot could learn natural motion patterns from long-term experience [11]–[13], and plan its path while anticipating people walking in learned directions even if it does not perceive any humans at a given moment. In other words, knowledge of the typical patterns of people movement could improve socially-compliant navigation by planning robot trajectories so that robots would follow the natural flows of people, and avoid congestions and areas where they would cause a nuisance. To address this, several authors [13]–[16] proposed models specifically aimed to represent the natural movement of people across the operational environment of the robot. These works aim at the spatial aspects of pedestrian flows, i.e., they represented the typical directions or velocities at different areas. However, the pedestrian flows themselves are not stationary, but, as shown in [16], [17], their intensities,

<sup>1</sup>Artificial Intelligence Center, Czech Technical University, {name.surname}@fel.cvut.cz

<sup>2</sup>Lincoln Centre for Autonomous Systems (L-CAS), University of Lincoln, UK, smolinamellado@lincoln.ac.uk

<sup>3</sup>Stanford University ransalu@stanford.edu

<sup>4</sup>Distributed Artificial Intelligence and Knowledge Laboratory (CIAD), University of Technology of Belfort-Montbéliard (UTBM), France, zhi.yan@utbm.fr

<sup>5</sup>AASS Mobile Robotics and Olfaction Lab, Örebro University, Sweden {name.surname}@oru.se

<sup>6</sup>University of Matej Bel in Banská Bystrica, Slovakia maria.stachova@umb.sk

The work is funded by CSF project 17-27006Y STRoLL, the CTU IGA grant No. SGS16/235/OHK3/3T/13, and FR-8J18FR018, PHC Barrande programme under grant agreement No. 40682ZH (3L4AV), and European Union’s Horizon 2020 research and innovation programme under grant agreement No. 732737 (ILIAD) and CZ grant CZ.02.1.01/0.0/0.0/16.019/0000765.

velocities, and directions strongly depend on time. A robot, capable of predicting future distributions of pedestrian flows, would be able to plan its collision-free, socially-compliant trajectories in advance, minimising the likelihood of having to alter its plan in order to avoid collisions.

We present a method capable of learning the natural flows of people and *how they change over time*. The core idea of the method is to model the time domain by several dimensions wrapped into themselves, which can efficiently represent periodicities of the pedestrian flow characteristics. Using a real-world dataset spanning over several weeks, we compare the method’s predictive performance to state of the art algorithms for pedestrian flow modelling. To promote reproducible and unbiased comparison, the dataset, code, and supporting materials publicly available [18], and the comparisons are performed using data provided by the authors of the methods mentioned above.

## II. RELATED WORK

The ability to autonomously move across space, i.e., navigation, is a pivotal competence of mobile robots. To navigate in an efficient, reliable and safe manner, a robot needs to be able to determine its position, position of its destination and it has to be able to plan its trajectory to avoid collisions. Both the accuracy of self-localisation and efficiency of the motion planning depend on the quality of the robot knowledge about its operational environment, i.e., on the fidelity and faithfulness of its internal representation of the surrounding world. Thus, a significant research effort was aimed at methods for building large-scale and accurate maps of the environment [19]. While most of the methods developed so far model the environment by static structures, deployment of robots in dynamic or changing environments raised the need to model the environment dynamics as well.

In the mapping and localisation community, the effects of the environment dynamics were studied mainly from the perspective of localisation reliability, which gradually deteriorates if the environment changes are neglected [20]. To deal with the changes, some approaches proposed to gradually adapt the maps by incrementally replacing their elements [4], by remapping the areas which changed [3], or by allowing multiple representations of the same location [2], by identifying the invariant characteristics of the world [21] or by general schemes to incrementally update continuous maps [22] using Bayesian techniques.

Another stream of the research proposed to exploit the observed dynamics to obtain more information about the environment. For example, Ambrus et al. [23] presented a method that can identify clusters of 3d data which changed between subsequent observations of the same location. Subsequent work demonstrated, that these clusters can be used for autonomous discovery of object models from the spatial changes observed [24].

Other researchers proposed to process the changes observed to obtain information about the temporal aspects of the long-term environment dynamics. For example, Dayoub et al. [25] and Rosen et al. [26] proposed to interpret the

changes in order to obtain models that would characterise the persistence of the environment states. The persistence models were then used to predict which elements of the environment should be used for localisation. The work of Tipaldi et al. [27] proposed to represent occupancy of grid cells by a Hidden Markov model, which also characterises the state persistence. Finally, the work of Krajník et al. [7] proposed to model the probability of environment states in the spectral domain, which captures cyclic (daily, weekly, yearly) patterns of environmental changes, which are often induced by humans. The concept of Frequency Map Enhancement (FreMEn) [7] was applied to a variety of scenarios, and was shown to improve both localisation [7], motion planning [28] and human-robot interaction [29].

Except for the FreMEn, the works mentioned above were aimed at the problem of localisation in environments undergoing a slow change. However, a substantial part of the natural environment dynamics is constituted by the rapid motion of people in the robot vicinity, which requires the robot to plan its trajectory with respect of the people around. As stated in [30], knowledge of the general pedestrian flows will allow the robots to move in a socially compliant manner, increasing not only their efficiency but also their acceptance by the public. To characterise the flows, the work of [13] proposes to extend an occupancy grid model by propagating information about the changes in the adjacent cells. Another discrete, grid-based model can be found in [14], where authors predict the paths of people based on an input-output Markov model associated with each cell. The authors of [31] assume the pedestrian flows change over time in a periodic fashion, and associate each cell of their grid with directional information enhanced by FreMEn [7].

Other streams of research tried to model the pedestrian movement by continuous representations. For example, in [32] authors show how to modify the plans of the robot motion by taking into account the long-term observations of people movement. However, this approach did not address the multimodality of pedestrian motion distribution making it unable to model motion in opposing directions. Later work [33] improved this particular aspect and presented a method that can model multimodal distributions of pedestrian movement directions. Kucner et al. also improved their approach in [13] and proposed a continuous representation in [34]. To model speed and direction of people, [35] introduced an expectation-maximisation scheme based on the Independent von Mises-Gaussian distributions [36]. They also showed that the model of the movement of people could be used to achieve more efficient navigation of the robot through human crowds [37].

Similarly, the work of [38] and [39] demonstrate that the incorporation of techniques to model periodic aspects of time into continuous spatial models results in powerful predictive representations. Furthermore, [16] shows the benefit of periodic temporal representations for pedestrian flow modelling. Therefore we propose using continuous spatiotemporal representations for modelling the pedestrian flows and how they change over time.

### III. METHOD DESCRIPTION

The aim of the proposed method is to find an estimation of Bernoulli distribution of an occurrence of spatio-direction-temporal events at time  $t_i$  on position  $x_i, y_i$  with the speed  $v_i$  at angle  $\phi_i$ . Since it is not possible to obtain multiple data with the same  $t_i$  by performing additional observations, one cannot calculate the Bernoulli distribution in a straightforward, frequentist way. This limitation is caused by the fact that the modelled events are sparse, and the process generating them is not stationary. To deal with the problem, we proposed in our previous works to use a “warped-hypertime” (WHyTe) projection of the timeline into a closed subset of multi-dimensional vector space, where each pair of dimensions would represent one periodicity [40]–[43]. Then, we create a model characterising the probability distribution of spatio-direction-temporal events in the vector space extended by the warped hypertime. To do so, we estimate distributions of the spatiotemporal events projected into the higher dimensional vector space using Expectation Maximisation algorithm for estimating Gaussian Mixture Models (EM GMM).

The idea behind the aforementioned projection is that events which occur with the same periodicity will form clusters in the hypertime space even if they are separated by long intervals of time. An intuitive example, shown in Figure 2 for the case of  $T = 1\text{day}$ , is that hypertime associates the given observations with the time of the day. Figure 2 shows an example output of a people detection system located in an office building corridor. Here, the detections overlap at mornings and evenings, when people leave and enter work, while the non-detections form clusters around noon and midnight, when people work or the building is vacant.

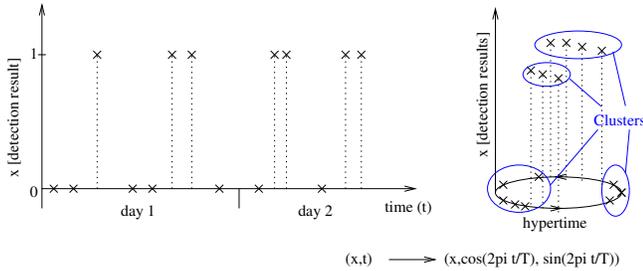


Fig. 2. Example of the warped hypertime projection on binary data (person detection) and one periodicity  $T$ . The numbers  $x_i$  observed at  $t_i$  are projected into a 3d vector space as  $(\mathbf{x}_i, \cos(2\pi t_i/T), \sin(2\pi t_i/T))$ , where they form clusters because they exhibit a periodic behaviour with a period  $T$ . The warped hypertime dimensions define a base of a cylinder, and values of  $x_i$  define a cylinder side.

#### A. Warped Hypertime Projection

Let us assume that the robot pedestrian tracking system provides us with vectors  $(x_i, y_i, v_i, \phi_i, t_i)$ , indicating the detected people positions, velocities and orientations as well as the timestamp of the observation. To avoid complications caused by the ambiguity of angles, we transform the aforementioned vector to  $(x_i, y_i, v_i \cos \phi_i, v_i \sin \phi_i, t_i)$  and denote it as  $(\mathbf{x}_i, t_i)$ .

Let us have a set of detections  $D(\mathbf{x}_i, t_i)$ ,  $i = 1 \dots n$  of occurrences and non-occurrences of some events at a location  $\mathbf{x}_i$  at time  $t_i$ , where  $D(\mathbf{x}_i, t_i) = 1$  for detected and  $D(\mathbf{x}_i, t_i) = 0$  for non-detected occurrences of the studied phenomenon. To determine the parameters of the warped hypertime projection, we need to identify the most distinctive temporal periodicities in the provided data. To do so, we create a time series  $R(t_i) = D(\mathbf{x}_i, t_i)$  by neglecting spatial components of the detections and apply the spectral decomposition method derived from the Frequency Map Enhancement [7]. First, we estimate the longest periodicity present in the data  $T_{max}$  and then we calculate  $\Upsilon$  periodicities as  $T_\tau = T_{max}/\tau$ , where  $\tau = 1 \dots \Upsilon$ . Then, we calculate the prominence of each periodicity as:

$$\gamma_\tau = \frac{1}{n} \sum_{i=1}^n R(t_i) e^{-j2\pi t_i/T_\tau}. \quad (1)$$

Since the experiments performed in [7] indicate that the most accurate predictions in human-populated environments are obtained by modelling 2-3 periodicities, we select two periodicities with the highest  $\gamma_\tau$  and denote them as  $T_{1,2}$ . Then, we project every measurement  $(\mathbf{x}_i, t_i)$  into the new vector space by:

$${}^@ \mathbf{x}_i = \left( \mathbf{x}_i, \cos \frac{2\pi t_i}{T_1}, \sin \frac{2\pi t_i}{T_1}, \cos \frac{2\pi t_i}{T_2}, \sin \frac{2\pi t_i}{T_2} \right). \quad (2)$$

#### B. Model of the probability distribution

We assume that the time-dependent occurrences of the phenomenon  $(\mathbf{x}_i, t_i)$  projected into the warped hypertime space as  ${}^@ \mathbf{x}_i$  are distributed in a way which allows modelling their distribution by Gaussian mixtures. To model the Bernoulli distribution of  $D(\mathbf{x}_i, t_i)$ , we split the dataset to occurrences and non-occurrences (these are mutually exclusive), and we build the mixture models of occurrences  $GMM_1({}^@ \mathbf{x}_i)$  and non-occurrences  $GMM_0({}^@ \mathbf{x}_i)$  in separate using an Expectation Maximisation algorithm. Thus, we obtain two models, characterised by cluster weights  $\alpha_{\{0,1\}j}$ , cluster centres  $\mathbf{c}_{\{0,1\}j}$  and covariances  $\Sigma_{\{0,1\}j}$ . These allow to determine the probability that a given projected sample  ${}^@ \mathbf{x}$  belongs to a particular cluster using a  $\chi^2$  distribution:

$$P_{\{0,1\}j} = 1 - P \left[ Q \leq (\mathbf{x} - \mathbf{c}_{\{0,1\}j})^T \Sigma_{\{0,1\}j}^{-1} (\mathbf{x} - \mathbf{c}_{\{0,1\}j}) \right], \quad (3)$$

where  $Q \sim \chi^2(d)$  and  $d$  is dimensionality of the constructed vector space. The overall probability  $M_{\{0,1\}}({}^@ \mathbf{x})$  of generating an occurrence of  ${}^@ \mathbf{x}$  by a mix of distributions  $GMM_{\{0,1\}}({}^@ \mathbf{x})$  is estimated as:

$$M_{\{0,1\}}({}^@ \mathbf{x}) = \sum_{j=1}^c \alpha_{\{0,1\}j} P_{\{0,1\}j}. \quad (4)$$

Then, the probability of the occurrence of  $(\mathbf{x}, t)$  is given by the following ratio based on its hypertime projection  ${}^@ \mathbf{x}$ :

$$M(\mathbf{x}, t) = \frac{M_1({}^@ \mathbf{x})}{M_1({}^@ \mathbf{x}) + M_0({}^@ \mathbf{x})}. \quad (5)$$

## IV. EVALUATION

### A. Dataset

The approach described above was evaluated using a dataset collected at the department of computer science at the University of Lincoln. The data recording was performed by a Pioneer 3-AT mobile robot equipped with a 3D lidar (Velodyne VLP-16) and a 2D lidar (Hokuyo UTM-30LX), using a reliable person detection method [44].

During the data collection, the robot remained stationary in a T-shaped junction, which allowed its sensors to scan the three connecting corridors simultaneously, covering a total area of around 75 m<sup>2</sup> (Fig. 3). However, since the robot could not stay at the corridor overnight due to safety rules, and it was needed by other researchers occasionally, we did not collect the data on a full 24/7 basis. Instead, the data collection was performed during ~10 hour long sessions starting before the usual working hours. Recharging of the batteries was performed overnight, where the building is vacant, and there are no people on the corridors.

The resulting dataset is composed of 9 data-gathering sessions recorded over four weeks. A typical session contains approximately 30000 detections of people walking in the monitored corridors. Every detection is represented by a vector  $(t, x, y, \phi, v)$  – the position, orientation, and speed of detected human in time. Similar to [45], we added 70000 “no detection” vectors of the positions, orientations, and speeds, where no human was detected (such as random vectors during the night, and people walking in the opposite direction than detected ones). As some of the methods in comparison do not model the speed, this value was set to  $v = 1.0$  for every measurement. For detailed information about particular methods used in the comparison, see section IV-C. The structured overview of the properties of individual methods can be seen in Table I.

The 3D lidar has 16 scan channels with a 360° horizontal and 30° vertical field-of-view, and was mounted at the height of 0.8 m from the floor on the top of the robot (Fig. 3 left), which allows us to have a perspective that covers the entire environment for data collection (Fig. 3 right). All people appearing in the corridor are detected and tracked in the 3D lidar’s frame of reference. More specifically, the 3D point cloud generated by the Velodyne lidar is first segmented into different clusters using an adaptive clustering method [44], then an offline trained SVM-based classifier was used for human classification. The 2D positions of the people are subsequently fed into a robust multi-target tracking system [46] using Unscented Kalman Filter (UKF) and Nearest Neighbour Joint Probability Data Association method (NNJPDA), and the human-like trajectories (in XY-plane) are eventually generated and recorded.

### B. Evaluation methodology

Following [47], we divided the dataset into a training and testing subset, where the training dataset consisted of seven days from three weeks, and the test dataset consisted of two days measured out of the time interval of the training dataset.

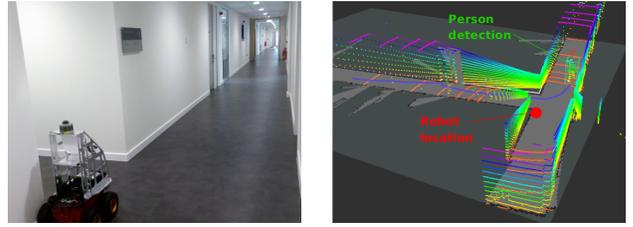


Fig. 3. Photo of the UoL dataset data collection setup: Robot location in the corridor and example of a person walking as seen by the 3D lidar.

We chose two different criteria to measure the quality of the model. The first is root-mean-square error (RMSE) [48] between model predictions  $M(\mathbf{x}_i, t_i)$  and test dataset values  $D(\mathbf{x}_i, t_i)$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (M(\mathbf{x}_i, t_i) - D(\mathbf{x}_i, t_i))^2}, \quad (6)$$

which is widely used in the time series forecasting [49].

The second criteria used is the level of similarity between human motion distributions, occurring at certain times and positions, obtained from the 2 test days and the ones predicted by the model. This metric is focused on how well the model can predict how a person would move in the case it is found, rather than how likely the robot is going to find one.

In order to do that, we have defined a spatial and temporal grid in order to cover the full map and the whole 2 test days. The different approaches can provide a probability motion distribution for any point in time/space, however, obtaining this same distribution with the ground truth data to make the comparison at a single time instance is not possible. The reason is that for an instance of time, we do not have enough detections in order to build a meaningful distribution. Instead, the idea is to compare the distribution obtained from the test data during a defined interval of time. In our evaluations, we have used a spatial grid, taking points every 1 meter in x and y directions, and 10 minutes long time intervals. During night time, when there are no detections, we assume equal probability for each orientation.

To make the comparison between the predicted and ground truth histograms for each interval and position, we have used the Chi-square distance. This distance indicates the level of similarity between two discrete distributions or histograms, so the higher the distance, the less accurate is our model prediction compared with the test data. The total Chi-square distance of the map for a single interval is defined as:

$$distance_{map} = \sum_{i=1}^n \sum_{b=1}^k \frac{(x_b - y_b)^2}{(x_b + y_b)}, \quad (7)$$

where  $n$  is the number of positions,  $k$  is the number of angular bins for the direction of people motion in the cells (in our case we have chosen  $k = 8$  taking the angles 0, 45, 90, 135, 180, 225, 270 and 315 degrees as values for each bin),  $x_b$  is the value of bin  $b$  in the predicted orientation histogram, and  $y_b$  is the value of the same bin  $b$  obtained from the ground truth.

### C. Methods compared in the experiment

1) *WHyTe*: There are two parameters, which affect the quality of WHyTe - the number of clusters  $c$  and the set of periodicities. The recent experiments showed that the number of clusters could be relatively small (usually up to 9) [43], and it seems, that the number of clusters is in relation with the topological structure of the space [42]. For this dataset from T-junction, we chose  $c = 3$  clusters. The second parameter can be derived from data iteratively, but recent experiments showed [42], [43], that the quality of prediction does not usually grow with more than 3 added hypertime circles. We selected the basic set of periodicities as proposed in [7] and found out that there were three strongly prominent components in the training data (six, twelve, and twenty-four hours), which we used in our method.

2) *STeF-Map*: STeF-Map [16], which stands for Spatio-Temporal Flow Map, is a representation that models the likelihood of motion directions on a grid-based map by a set of harmonic functions, which capture long-term changes of crowd movements over time. The underlying geometric space is represented by a grid, where each cell contains  $k$  temporal models, corresponding to  $k$  discretised orientations of people motion through the given cell over time. Since the total number of temporal models, which are of a fixed size, is  $k \times n$  where  $n$  is the total number of cells, the spatiotemporal model does not grow over time regardless of the duration of data collection. The temporal models, which can capture patterns of people movement, are based on the FreMEn framework [7]. FreMEn is a mathematical tool based on the Fourier Transform, which considers the probability of a given state as a function of time and represents it by a combination of harmonic components. The idea is to treat a measured state as a signal, decompose it using the Fourier Transform, and obtain a frequency spectrum with the corresponding amplitudes, frequencies and phase shifts. Then, transferring the most prominent spectral components to the time domain provides an analytic expression representing the likelihood of that state at a given time in the past or future.

This model assumes that it is provided with people detection data, comprising the person position, orientation and timestamp of the detection  $(x, y, \alpha, t)$ . When building the model, the  $x, y$  positions are discretised and assigned to the corresponding cell, and the orientation  $\alpha$  is assigned to one of the  $k$  bins, whose value is incremented by 1. After a predefined interval of time, the bins values are normalised, and the results are used to update the spectra of the temporal models. Then, the bin values are reset to 0, and the counting starts again.

In order to retrieve the behaviour of human movement through a given cell at a certain time  $t$  (which can be at the future or at the past), the likelihood for each discretised orientation associated with a cell can be computed as:

$$p_\theta(t) = p_0 + \sum_{j=1}^m p_j \cos(\omega_j t + \varphi_j), \quad (8)$$

where  $p_0$  is the stationary probability,  $m$  is the number of the most prominent spectral components, and  $p_j$ ,  $\omega_j$  and

$\varphi_j$  are their amplitudes, periods and phases. The spectral components  $\omega_j$  are drawn from a set of  $\omega_s$  that covers periodicities ranging from 14 h to 1 week with the following distribution:

$$\omega_s = \frac{3600 \cdot 24 \cdot 7}{s}, \quad s \in 1, 2, 3, \dots, 12. \quad (9)$$

3) *Directional grid maps*: Directional grid maps (DGM) [50] are designed to model the *directional uncertainty* of dynamic environments. The inputs to the model are directions of objects at different locations of the environment, and the outputs are a set continuous probability density functions indicating most probable directions dynamic objects move at various locations of the environment. In order to build a DGM, firstly, the 2D or 3D environment is divided into a fixed-sized grid. Then, a mixture of von Mises distribution is assigned to each cell to model the multimodal angular uncertainty. Analogous to a Gaussian distribution, however with a limited  $[-\pi, +\pi]$  support, a von Mises distribution is controlled by its mean angle and concentration (inverse variance) parameters. The number of von Mises components for each mixture is determined by the number of density-wide clusters using the DBSCAN algorithm. Having initialised the von Mises distributions with the cluster centres, the parameters are learned using Expectation-Maximization (EM). In experiments, it takes 1 to 4 iterations to converge the EM. Since the directional grid maps are not designed to deal with the spatiotemporal domain with periodic patterns, in this experiment, the temporal domain is also discretised every 15 minutes in addition to the  $2 \text{ m} \times 2 \text{ m}$  spatial discretisation. For this experiment, as a proxy, we attempt to estimate the people density by considering the cells where the initial set of mixture parameters changes with time. Therefore, the proxy count probabilities are always either 0 or 1 and not the exact people density. In the future, it is possible to replace the von Mises distribution with a Gaussian distribution or replace the Bernoulli likelihood in [22] with a Gaussian likelihood to accurately model such spatial density estimations.

4) *CLiFF-Map Model*: Circular Linear Flow Field map (CLiFF-map) [35] is a technique for encoding patterns of movement as a field of Gaussian mixtures. They can be combined with semi-wrapped Gaussian mixture models (SWGMM) to model multi-modal motion.

$$p(\mathbf{V}|\boldsymbol{\xi}) = \sum_{j=1}^J \pi_j \mathcal{N}_{\boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j}^{\mathcal{S}\mathcal{W}}(\mathbf{V}) \quad (10)$$

with  $\sum_{j=1}^J \pi_j = 1$ .

This uses a semi-wrapped normal distribution, distributed along the circumference and height of a cylinder. It is represented as a semi-wrapped normal distribution. It can be derived from:

$$\mathcal{N}_{\boldsymbol{\mu}, \boldsymbol{\Sigma}}^{\mathcal{S}\mathcal{W}}(\mathbf{V}) = \sum_{k \in \mathbb{Z}} \mathcal{N}_{\boldsymbol{\mu}, \boldsymbol{\Sigma}} \left( \begin{bmatrix} \theta \\ \rho \end{bmatrix} + 2\pi \begin{bmatrix} k \\ 0 \end{bmatrix} \right). \quad (11)$$

where  $\mathbf{V} = (\theta, \rho)$  represents the instantaneous velocities,  $\theta \in [0, 2\pi)$  is the direction, and  $\rho \in \mathbb{R}^+$  the speed.

TABLE I  
QUALITATIVE COMPARISON OF METHODS

Name	Method	Time		Representation					Complexity	
		References	long-term	short-term	time	space	intensity	direction	speed	Memory [kB]
WHyTe	[40]	✓	×	C	C	C	C	C	2	60
STeF	[16]	✓	×	C	D	×	D	×	140	20
DGM	[50]	✓	×	D	D	C	C	×	20	72
CLiFF	[35]	×	×	×	D	C	C	C	6k	10 <sup>4</sup>
LSTM	[51]	×	✓	C	C	C	C	C	900	10 <sup>6</sup>

Note 1: In the ‘Representation’ columns, C stands for the continuous, and D for the discrete representation of variables provided by the method.

Note 2: CLiFF-map was developed using Matlab and other methods are based on the Python language.

5) *LSTM*: We also implemented a deep-learning model for a point of comparison. A long short-term memory [51] neural network was built using Keras atop of the TensorFlow library. It consisted of 4 layers of 50 LSTM units followed by a fully connected layer with 72000 trainable parameters. It was then trained to convergence on the training set. It is important to note, however, that this method consumed significantly more computing power, both during training and prediction.

#### D. Evaluation results

The results of the evaluation, which are summarised in Table II, indicate that WHyTe achieved the lowest root mean squared error, but for the  $\chi^2$  distance, STeF present the lower score. We believe that the reason behind that is because STeF represents each orientation in each position by its own temporal model. Although DGM is not designed to estimate the probability of occurrence and it only returns 0 or 1, and therefore RMSE is not a proper measurement for this method, it was comparable with WHyTe in  $\chi^2$  statistics.

To fit the evaluation procedure, CLiFF-map was discretised into eight orientation bins. CLiFF-map predicts directions on specific positions better than WHyTe during the day, but as this model does not take into account the temporal dimension, its night results are worse.

Low values of RMSE in Table II show that WHyTe can accurately model both movement directions and human occurrences. Modelling the joint probability of occurrences and directions is the crucial property of the WHyTe, which is supposed to provide apriori knowledge of the dynamics of human-populated environments to the autonomous robots.

We also include an LSTM model as it is commonly considered a state-of-the-art method for temporal predictions. The LSTM model was trained using four NVIDIA Tesla V100 SXM2 32GB for 10 hours, and its model size was 900 KiB. However, as the LSTM is tailored for short-term predictions (compared to the prediction horizons of STeF or WHyTe), its predictions quickly converge to the mean probability of people directions across the entire training dataset (both spatially and temporally). Therefore, LSTM predicts that in a long-term horizon, the distribution of people

TABLE II  
PREDICTION ERRORS OF THE EVALUATED MODELS AND DATASETS

Testing sets Criterion	Days		Nights		Days and nights	
	RMSE	$\chi^2$	RMSE	$\chi^2$	RMSE	$\chi^2$
WHyTe	<b>0.50</b>	23.4	<b>0.00</b>	0.2	<b>0.40</b>	23.6
STeF	0.57	<b>10.6</b>	0.02	8.1	0.46	<b>18.7</b>
DGM	0.70	25.5	0.83	<b>0.0</b>	0.75	25.5
CLiFF	0.60	15.5	0.16	9.2	0.50	24.7
LSTM	0.57	25.5	0.22	<b>0.0</b>	0.48	25.5

movement directions will be uniform. DGM, as well as WHyTe, predict very low probabilities of people presence during the night, which corresponds to uniform distribution of walking directions as well. Thus, the  $\chi^2$  metric for these three methods for the night data is lower compared to STeF and CLiFF.

WHyTe, STeF and DGM were developed using Python language, their training on regular personal notebooks lasted about one minute, and the model sizes are 2 KiB, 140 KiB, and 20 KiB respectively, which indicates, that they could be applied in real robotic tasks. It should be noted, that model created by WHyTe is smaller by magnitude(s) to its competitors, which is an essential attribute for building models over large areas, see Table I.

#### V. CONCLUSION

We propose an approach capable of representing pedestrian flows and how they change over time. Unlike methods, that perform predictions based on recent observations, the model presented can predict the pedestrian flow intensity and direction using observations gathered days to weeks before the prediction. Instead of short-term predictions based on actual observations, which force the robot to react to the current situation in a suboptimal manner, a robot utilising our method would be able to anticipate human presence and movement direction from long-term observations and plan its trajectory to minimise the need to evade people. This helps to avoid situations, where the robot interferes with the natural pedestrian flows in its operational area, allowing seamless, socially-acceptable navigation in human-populated environments.

The proposed representation is based on the idea of warped hypertime (WHyTe), which projects the time into a constrained subset of a multidimensional vector space, constructed to reflect the patterns of human habits.

We evaluated the presented method on a real dataset, gathered over four weeks and compared its predictive accuracy to state-of-the-art methods provided by their authors. Two criteria were used: RMSE, which reflects the ability to predict the joint probability of the presence and the direction of pedestrian movement, and  $\chi^2$  statistics, that reflects the ability to predict the conditional probability of the movement direction given that a human is present at some specific position and time. Although the proposed method was not able to compete with STeF and CLiFF methods in  $\chi^2$  statistics, it achieved the best prediction in terms of RMSE. This indicates that while the method is not as good in

predicting the flow directions, it has a superior performance in predicting intensities of the pedestrian flows. Moreover, we showed that our method is by the magnitude(s) smaller compared to the other ones, indicating its suitability to model large-scale environments.

In the future, we will evaluate the impact of the methods used in this comparison on the ability of robots to generate collision-free trajectories in advance.

The WHyTe code, dataset, and other materials used for our experiments are available at [18].

## REFERENCES

- [1] P. Biber and T. Duckett, "Experimental analysis of sample-based maps for long-term slam," *The Int. Journal of Robotics Research*, 2009.
- [2] W. Churchill and P. Newman, "Experience-based navigation for long-term localisation," *The Int. Journal of Robotics Research*, 2013.
- [3] K. Konolige and J. Bowman, "Towards lifelong visual maps," in *IEEE/RSJ Int. Conf. on Intelligent Robots and Systems (IROS)*, 2009.
- [4] S. Hochdorfer and C. Schlegel, "Towards a robust visual slam approach: Addressing the challenge of life-long operation," in *International Conference on Advanced Robotics*, 2009.
- [5] D. Tipaldi, D. Meyer-Delius, and W. Burgard, "Lifelong localization in changing environments," *Int. Journal of Robotics Research*, 2013.
- [6] D. M. Rosen, J. Mason, and J. J. Leonard, "Towards lifelong feature-based mapping in semi-static environments," in *ICRA*. IEEE, 2016.
- [7] T. Krajník, J. P. Fentanes, J. M. Santos, and T. Duckett, "Fremer: Frequency map enhancement for long-term mobile robot autonomy in changing environments," *IEEE Transactions on Robotics*, 2017.
- [8] B. Song, W. Chen, J. Wang, and H. Wang, "Long-term visual inertial slam based on time series map prediction," in *IROS*, 2019.
- [9] D. Hebesberger *et al.*, "A long-term autonomous robot at a care hospital: A mixed methods study on social acceptance and experiences of staff and older adults," *Int. Journal of Social Robotics*, 2017.
- [10] L. Sun *et al.*, "3dof pedestrian trajectory prediction learned from long-term autonomous mobile robot deployment data," in *ICRA*, 2018.
- [11] M. Bennewitz, W. Burgard, G. Cielniak, and S. Thrun, "Learning motion patterns of people for compliant robot motion," *The International Journal of Robotics Research*, vol. 24, no. 1, pp. 31–48, 2005.
- [12] D. Vasquez, T. Fraichard, and C. Laugier, "Growing hidden markov models: An incremental tool for learning and predicting human and vehicle motion," *The Int. Journal of Robotics Research*, 2009.
- [13] T. Kucner, J. Saarinen, M. Magnusson, and A. J. Lilienthal, "Conditional transition maps: Learning motion patterns in dynamic environments," in *IROS*, 2013.
- [14] Z. Wang, R. Ambrus, P. Jensfelt, and J. Folkesson, "Modeling motion patterns of dynamic objects by iohmm," in *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, 2014.
- [15] C. S. Swaminathan *et al.*, "Down the cliff: Flow-aware tralatory planning under motion pattern uncertainty," in *IROS*, 2018.
- [16] S. Molina, G. Cielniak, and T. Duckett, "Go with the flow: Exploration and mapping of pedestrian flow patterns from partial observations," in *Proc. of Int. Conference on Robotics and Automation (ICRA)*, 2019.
- [17] D. Bršćic, T. Kanda, T. Ikeda, and T. Miyashita, "Person tracking in large public spaces using 3-d range sensors," *IEEE Transactions on Human-Machine Systems*, vol. 43, no. 6, pp. 522–534, 2013.
- [18] T. Krajník *et al.*, "Chronorobotics code and dataset repository," <http://chronorobotics.tk>.
- [19] C. Cadena *et al.*, "Past, present, and future of simultaneous localization and mapping: Toward the robust-perception age," *Transactions on robotics*, vol. 32, no. 6, pp. 1309–1332, 2016.
- [20] L. Kunze, N. Hawes, T. Duckett, M. Hanheide, and T. Krajník, "Artificial intelligence for long-term robot autonomy: a survey," *IEEE Robotics and Automation Letters*, vol. 3, no. 4, pp. 4023–4030, 2018.
- [21] S. Lowry and M. J. Milford, "Supervised and unsupervised linear learning techniques for visual place recognition in changing environments," *Transactions on Robotics*, vol. 32, no. 3, pp. 600–613, 2016.
- [22] R. Senanayake and F. Ramos, "Bayesian hilbert maps for dynamic continuous occupancy mapping," in *Conf. on Robot Learning*, 2017.
- [23] R. Ambrus, N. Bore, J. Folkesson, and P. Jensfelt, "Meta-rooms: Building and maintaining long term spatial models in a dynamic world," in *Int. Conf. on Intelligent Robots and Systems (IROS)*, 2014.
- [24] T. Faeulhammer *et al.*, "Autonomous learning of object models on a mobile robot," *Robotics and Automation Letters*, 2016.
- [25] F. Dayoub, G. Cielniak, and T. Duckett, "Long-term experiments with an adaptive spherical view representation for navigation in changing environments," *Robotics and Autonomous Systems*, 2011.
- [26] D. M. Rosen, J. Mason, and J. J. Leonard, "Towards lifelong feature-based mapping in semi-static environments," in *ICRA*, 2016.
- [27] G. D. Tipaldi *et al.*, "Lifelong localization in changing environments," *The International Journal of Robotics Research*, 2013.
- [28] J. P. Fentanes *et al.*, "Now or later? predicting and maximising success of navigation actions from long-term experience," in *International Conference on Robotics and Automation (ICRA)*. IEEE, 2015.
- [29] M. Hanheide *et al.*, "The When, Where, and How: An Adaptive Robotic Info-Terminal for Care Home Residents A long-term Study," in *ACM/IEEE Int. Conf. on Human-Robot Interaction (HRI)*, 2017.
- [30] T. P. Kucner, "Probabilistic mapping of spatial motion patterns for mobile robots," Ph.D. dissertation, Örebro University, 2018.
- [31] S. Molina, G. Cielniak, T. Krajník, and T. Duckett, "Modelling and predicting rhythmic flow patterns in dynamic environments," in *Annual Conference Towards Autonomous Robotic Systems*, 2018.
- [32] S. T. O'Callaghan, S. P. N. Singh, A. Alempijevic, and F. T. Ramos, "Learning navigational maps by observing human motion patterns," *International Conference On Robotics And Automation (ICRA)*, 2011.
- [33] L. McCalman, S. O'Callaghan, and F. Ramos, "Multi-modal estimation with kernel embeddings for learning motion models," in *Int. Conference on Robotics and Automation (ICRA)*. IEEE, 2013.
- [34] T. Kucner *et al.*, "Tell me about dynamics!: Mapping velocity fields from sparse samples with semi-wrapped gaussian mixture models," in *RSS 2016 Workshop: Geometry and Beyond-Representations, Physics, and Scene Understanding for Robotics*, 2016.
- [35] T. P. Kucner *et al.*, "Enabling flow awareness for mobile robots in partially observable environments," *Robotics and Autom. Letters*, 2017.
- [36] A. Roy, S. K. Parui, and U. Roy, "A mixture model of circular-linear distributions for color image segmentation," *International Journal of Computer Applications*, vol. 58, no. 9, 2012.
- [37] L. Palmieri, T. P. Kucner, M. Magnusson, A. J. Lilienthal, and K. O. Arras, "Kinodynamic motion planning on gaussian mixture fields," in *Int. Conference on Robotics and Automation (ICRA)*. IEEE, 2017.
- [38] R. Senanayake, O. Simon Timothy, and F. Ramos, "Predicting spatio-temporal propagation of seasonal influenza using variational gaussian process regression," in *AAAI*, 2016, pp. 3901–3907.
- [39] A. Tompkins and F. Ramos, "Fourier feature approximations for periodic kernels in time-series modelling," in *AAAI Conference on Artificial Intelligence*, 2018.
- [40] T. Vintř, K. Eyisoy, and T. Krajník, "A practical representation of time for the human behaviour modelling," *Forum Statisticum Slovacum*, vol. 14, pp. 61–75, 2018.
- [41] T. Vintř *et al.*, "Spatiotemporal models of human activity for robotic patrolling," in *Int. Conf. on Modelling and Simulation for Autonomous Systems*. Springer, 2018, pp. 54–64.
- [42] T. Vintř, Z. Yan, T. Duckett, and T. Krajník, "Spatio-temporal representation for long-term anticipation of human presence in service robotics," in *International Conference on Robotics and Automation (ICRA)*. IEEE, 2019.
- [43] T. Krajník *et al.*, "Warped hypertime representations for long-term autonomy of mobile robots," *Robotics and Automation Letters*, 2019.
- [44] Z. Yan, T. Duckett, and N. Bellotto, "Online learning for human classification in 3d lidar-based tracking," in *IROS*, 2017.
- [45] S. T. O'Callaghan and F. T. Ramos, "Gaussian process occupancy maps," *The International Journal of Robotics Research*, 2012.
- [46] N. Bellotto and H. Hu, "Computationally efficient solutions for tracking people with a mobile robot: an experimental evaluation of bayesian filters," *Autonomous Robots*, vol. 28, pp. 425–438, 2010.
- [47] M. Oliveira, L. Torgo, and V. S. Costa, "Evaluation procedures for forecasting with spatio-temporal data," in *Joint European Conf. on Machine Learning and Knowledge Discovery in Databases*, 2018.
- [48] R. J. Hyndman and A. B. Koehler, "Another look at measures of forecast accuracy," *International journal of forecasting*, 2006.
- [49] R. J. Hyndman and G. Athanasopoulos, *Forecasting: principles and practice*. OTexts, 2018.
- [50] R. Senanayake and F. Ramos, "Directional grid maps: modeling multimodal angular uncertainty in dynamic environments," in *IEEE/RSJ Int. Conference on Intelligent Robots and Systems (IROS)*, 2018.
- [51] S. Hochreiter and J. Schmidhuber, "Long short-term memory," *Neural computation*, vol. 9, no. 8, pp. 1735–1780, 1997.