Incorporating Ego-motion Uncertainty Estimates in Range Data Registration

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***Abstract*— Local scan registration approaches commonly only utilize ego-motion estimates (e.g. odometry) as an initial pose guess in an iterative alignment procedure. This paper describes a new method to incorporate ego-motion estimates, including uncertainty, into the objective function of a registration algorithm. The proposed approach is particularly suited for feature-poor and self-similar environments, which typically present challenges to current state of the art registration algorithms. Experimental evaluation shows significant improvements in accuracy when using data acquired by Automatic Guided Vehicles (AGVs) in industrial production and warehouse environments.**

I. Introduction

Registration — the problem of determining the relative pose between sensory data; is one of the fundamental building blocks of autonomous mapping and localization systems. Several approaches have been suggested in the past, and examples that are prominently used in robotics contexts include ICP [1], [2], NDT [3], [4], [5], NICP [6], and GICP [7]. A shortcoming inherent to all the aforementioned methods is that any additional information – for example, obtained by odometry – is solely incorporated as an initial estimate for the registration method and is not exploited further, *during* the registration process. There are approaches that utilizes initial estimates along with an uncertainty estimate to improve correspondences in scan matching, for example, point to point correspondences [8] and to filter out spurious readings due noise or moving objects [9]. For vision based approaches it is common to have a tightly coupled integration with egomotion sensing, see for example [10].

Looking at a complete mapping solution, such as approaches based on graph-SLAM, incremental pose estimates are commonly added into the graph structure which is then further optimized. Incremental tracking and mapping approaches [11], [12] do not typically utilize this information, which can result in inconsistencies in feature-poor environments.

In general, local registration methods have problems with self similar and feature-poor environments: for example, corridors where registration methods commonly underestimate the distance traveled. In this work we propose a method to integrate information about the expected uncertainty of the initial guess into the scan registration algorithm, in order to improve the accuracy of registration in feature-poor environments. We build upon an existing state of the art point cloud registration method — the NDT distribution-to-distribution (NDT-D2D [5]) algorithm; and modify it to use available ego-motion estimates within the registration procedure. We incorporate the uncertainty of the ego motion estimate and use it to penalize scan alignment solutions which are inconsistent with the initial guess. Although we utilize some properties inherent to the NDT-D2D registration approach, the ideas presented in this work could equally well be applied to augment other local registration methods.

The main contribution of this work is thus an augmented registration algorithm which integrates multiple redundant sources of information in the scan alignment process. We demonstrate that our approach can be used to enable an open-loop tracking and mapping system [12] to build consistent local submaps, even in feature-poor environments. As a direct application of our approach, we envision that locally consistent submaps with a global graph-SLAM back-end would entail several advantages over current state of the art keyframe-based systems. First, this strategy would substantially reduce the number of graph nodes in the global optimization scheme and allow scalability to much larger environments. And second, using a dynamics-aware submap representation ($e.g.$, NDT-OM [13]) will allow to re-construct more consistent global maps and to handle dynamic entities by fusing multiple sensory readings rather than handling dynamics and spurious data in the registration.

II. Methods

The robotics and computer graphics communities have proposed a multitude of scan registration methods, a thorough review of which is outside the scope of this work. We will, however, discuss in slightly more detail registration methods from the Normal Distributions Transform (NDT) [3] family, since one of them (NDT-D2D [5]) serves as a base for our approach. For a more detailed comparison between NDT-based registration methods and the state of the art, we refer to a recent standard data set-based comparison [14].

A. NDT-D2D

The central idea of the NDT representation is to model the observed range points using a set of Gaussian probability distributions. Given a point set, its NDT model is created by discretising space using a regular grid and fitting a Gaussian probability density function $\mathcal{N} = \{C_i, \boldsymbol{\mu}_i\}$ to the samples in each voxel. Magnusson et al. [4] applied the NDT

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Fig. 1: Top: NDT map illustration from an incremental pose estimation test in a warehouse. Bottom: Picture from one of the aisles in the warehouse.

representation to the domain of 3D scan registration. The central idea in their approach is to maximize the likelihood of points from one range scan, given the NDT model created from a previously known reference 3D scan (NDT-P2D). Stoyanov et al. [5] propose an extension of the registration approach — the NDT Distribution-to-Distribution (D2D) algorithm, which minimizes the sum of L_2 distances between pairs of Gaussian distributions. Formally, the transformation between two point sets $\mathcal{M}$ and $\mathcal{F}$ is found by minimizing:

$$f_{D2D}(\boldsymbol{p}) = \sum_{i=1,j=i}^{n_{\mathcal{M}},n_{\mathcal{F}}} -d_1 \exp\left(-\frac{d_2}{2}\boldsymbol{\mu_{ij}}^T(R^T C_i R + C_j)^{-1}\boldsymbol{\mu_{ij}}\right) \tag{1}$$

over the transformation parameters $\boldsymbol{p}$, where: $n_{\mathcal{M}}$ and $n_{\mathcal{F}}$ are the number of Gaussian components in the NDT models of $\mathcal{M}$ and $\mathcal{F}$; R and $\boldsymbol{t}$ are the rotation and translation components of $\boldsymbol{p}$; $\boldsymbol{\mu}_i, C_i$ are the mean and covariance of each Gaussian component; $\boldsymbol{\mu_{ij}} = R\boldsymbol{\mu_i} + \boldsymbol{t} - \boldsymbol{\mu_j}$ is the transformed mean vector distance; and d_1, d_2 are regularization factors (fixed values of $d_1 = 1$ and $d_2 = 0.05$ were used). The optimization over $\boldsymbol{p}$ can be done efficiently using Newton method optimization with analytically computed derivatives.

If egomotion estimates, for example odometry, are available they are typically used as a starting point for the optimization procedure: $i.e.$, to initialize the transformation parameters R and $\boldsymbol{t}$ associated to an initial guess $\boldsymbol{p_0}$. The success of the registration procedure then relies on $\boldsymbol{p_0}$ being within the convergence basin of a globally optimal solution $\boldsymbol{p^*} \in \arg\min_{\boldsymbol{p}} f_{D2D}(\boldsymbol{p})$. Providing initial transformation parameters does not by any means change the shape of the objective function though. For example, in a corridor with limited amount of features this optimization procedure will tend to maximize the overlap between the two observations, which would typically cause the translation to be underestimated. The reason is that two sensory readings from a featureless corridor environment will be very similar, resulting in a global minimum of $f_{D2D}(\boldsymbol{p})$ for $\boldsymbol{p^*}$ close to 0, which may not reflect the information contained in the initial guess $\boldsymbol{p_0}$ from egomotion estimation. In order to improve the accuracy of registration in such conditions, it would clearly be beneficial to integrate the initial guess (and our degree of confidence in its accuracy) in the objective function. In the next subsection, we propose our approach to incorporating an egomotion estimate in the registration objective.

B. NDT-DTD with Soft Constraints on Pose

One method to actively incorporate a (incremental) pose estimate to the registration method described above is to constrain the space of feasible solutions by imposing additional inequality constraints. For example, one could instead solve the problem:

$$\begin{aligned} &\underset{\boldsymbol{p}}{\text{minimize}}\ f_{D2D}(\boldsymbol{p}) \\ &\text{subject to}\ ||\boldsymbol{p} - \boldsymbol{p_0}||_2 \leq \boldsymbol{b}, \end{aligned} \tag{2}$$

which would constrain the solution $\boldsymbol{p^*}$ within a neighbourhood of the initial guess $\boldsymbol{p_0}$. Such a solution would however disregard the information on the reliability of the egomotion estimate, which is typically readily available through the estimate covariance Σ. To incorporate that information, we could instead use the Mahalanobis distance under the covariance Σ:

$$\begin{aligned} &\underset{\boldsymbol{p}}{\text{minimize}}\ f_{D2D}(\boldsymbol{p}) \\ &\text{subject to}\ (\boldsymbol{p} - \boldsymbol{p_0})^T\Sigma^{-1}(\boldsymbol{p} - \boldsymbol{p_0}) \leq \boldsymbol{1}, \end{aligned} \tag{3}$$

where we have eliminated the free parameter $\boldsymbol{b}$ by assuming it is encoded in the covariance Σ. The so formulated problem could be solved by a constrained non-convex optimization approach. We argue however that this is neither necessary, nor desirable: it is much more prudent to allow the registration method to violate the constraints imposed by odometry in cases when strong features are present in the environment. Therefore, we propose to incorporate the inequality constraint into the objective as a penalty term. We re-formulate the problem as:

$$\underset{\boldsymbol{p}}{\text{minimize}}\ f_{D2D}(\boldsymbol{p}) + \lambda(\boldsymbol{p} - \boldsymbol{p_0})^T\Sigma^{-1}(\boldsymbol{p} - \boldsymbol{p_0}), \tag{4}$$

where λ is a penalty coefficient (set to 1 in our experimental evaluation). Please note that for the specific objective function used for the soft constraints the λ term should be 1 since we here maximize the sum of two likelihoods.

To model the covariance Σ, we adopt the motion model originally presented by Eliazar et al. [15], where the uncertainty in each incremental motion step is modeled through a set of normal distributions – D, C, and T; with D and C associated to the forward and lateral motion components, and T to the rotation. The covariance matrix for an incremental motion step ($\Delta\mathbf{x} = (\Delta x, \Delta y, \Delta\theta)$), in the vehicles frame, is computed as:

$$\begin{pmatrix} d^2 D_d + t^2 D_t & 0 & 0 \\ 0 & d^2 C_d + t^2 C_t & 0 \\ 0 & 0 & d^2 T_d + t^2 T_t, \end{pmatrix} \tag{5}$$

where $d = \sqrt{\Delta x^2 \Delta y^2}$ and $t = |\Delta\theta|$. Although the egomotion estimate and its associated covariance assume a 2D planar world (3DoF), the registration algorithm is still optimizing over the full 6DoF pose space. As a future extension of our approach, we plan to integrate multiple "soft" constraints that originate from other types of egomotion sensors, which potentially could also be in a global reference frame (such as roll or pitch from an IMU). In the experimental section the variance of Σ in (4) for the roll, pitch and z directions is set to a constant value ($= 1.$).

III. Evaluation

In order to evaluate the proposed approach, we used data from two different industrial environments: a dairy production facility (Fig. 2) and a warehouse (Fig. 2). The data were collected using already installed automatic guided vehicles (AGVs) used in production. The data consist of 3D laser range measurements acquired from a Velodyne HDL-32 mounted one of the vehicles, odometry obtained from the wheel encoders (steer and drive) of the vehicle, as well as pose estimates from a commercial reflector based global localization system. The reflector based localization system utilizes pre-installed markers and provides an accuracy of < 0.02 meters and is used throughout the evaluations as ground truth. In the dairy production facility and in one of the two warehouse datasets (the "loop" dataset) the AGV was in autonomous operation mode, whereas in the second warehouse dataset (the "zig-zag aisle" dataset) the AGV truck was manually operated (see Fig. 3).

The evaluation is conducted by evaluating the registration approach alone as well as utilizing the simultaneous mapping and tracking approach detailed in [12]. This approach incrementally builds a map by registering the current sensory data with the map, and then updating the map by integrating the measurements from the scan. A more detailed description on the measurement updates of the underlying NDT-OM map can be found in [13]. Evaluating the registration method as a component of a mapping and tracking system allows for a comparison under more realistic usage setting, compared to scan to scan matching in isolation.

The parameters that are varied in the evaluations are the resolution of the NDT maps ($i.e.$, the regular grid cell size), the sensor cut-off distance (which limits the sensor range and therefore provides less information), and whether soft-constraints are used or not. The motion parameters used in the experiments, unless otherwise specified, is $D_d = 0.004$, $D_t = 1$, $C_d, C_t, T_d, T_d = 100$. The parameters are selected in order to place a large weight on the part of the egomotion estimate that corresponds to distance traversed.

In addition to the real world environments a simulated environment consisting of an "endless" corridor (Sec. III-A) is used to illustrate the approach.

The proposed registration approach will be available in the next release of the perception_oru ROS package[1], while the datasets used for evaluation will be made publicly available[2].

[1]http://wiki.ros.org/perception_oru

[2]http://aass.oru.se/Research/Learning/datasets.html

A. Endless corridor - simulated environment

To illustrate the behavior of the registration approaches in an environment without any features that allow to reliably determine a translation component, a dataset was acquired from a simulated gazebo environment "endless corridor" (see Fig. 4). A vehicle with a combined steer and drive wheel kinematic, similar to that of a real forklift AGV, was used to collect data from the environment. Odometry was obtained in the same way as for the real vehicle: by integrating the rotational velocity of the drive wheel, combined with the steering angle at each time step. Odometry, ground truth (obtained by querying the simulation engine) and range data were recorded.

The results are depicted in Fig. 4. It is clear that aligning the simulated laser data using the extended registration version with soft constraints yields better results. However, by looking at the rotational part in the relative pose error (RPE) on this dataset the mean error is 0.021 ± 0.013 degrees and 0.053 ± 0.046 without and with soft constraints respectively. The larger error using soft constraints could here be explained by the fact that the additional flexibility for the registration was limited by the added cost of the egomotion based constraint. The relative position error is 0.542 ± 0.552 and 0.009 ± 0.005 without and with soft constraints respectively.

B. Motion parameters selection

The motion parameters in our covariance model directly affect the shape of the registration objective function. Therefore, before continuing with the full evaluation, we evaluate the sensitivity of our approach to selecting different D_d values — $i.e.$, we vary the motion parameter that influences the variance in the direction of forward motion. To do so, we ran the tracker on the "zig-zag" dataset from the warehouse environment (see Fig. 3) and compared the Absolute Translational Errors (ATEs) against the ground truth, for different values of D_d. The results are shown in Fig. 5. What we can observe is that the selection of the D_d parameter is more important for a lower sensor cutoff distance and reasonable parameters lie in the range of 0.001 to 0.01. Based on this the D_d^2 parameter were set to 0.004 throughout all subsequent evaluations.

C. Warehouse

One of the motivations of the proposed approach is to provide additional information for registration in feature-poor environments. To evaluate the suitability of our method, we use two data sets from a logistics warehouse consisting of multiple rows of shelves and aisles: one dataset records zig-zag driving between different aisles; while the second dataset traces a large loop (see left and middle subfigures in Fig. 3). The evaluation is performed by varying both the sensor cut-off distance and the resolution of the NDT map, and the resulting ATE plots are shown in Fig. 6 and 7. For both datasets we observe that the original NDT-D2D registration method results in a drastic degradation of performance at resolutions of about 0.8 meters, whereas the performance

Fig. 2: Environments: Dairy production environment, production area (top) and fridge storage area (middle). Warehouse environment (right) with high-storage shelves and long corridors.

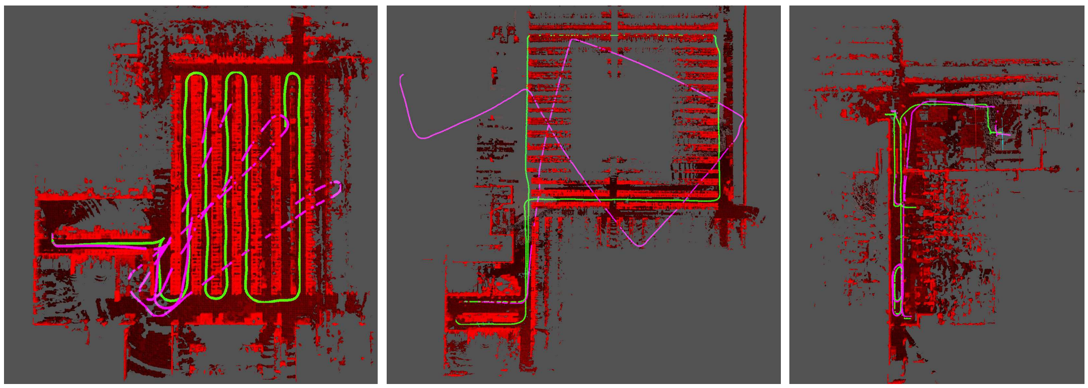

Fig. 3: Different datasets used in the different evaluations depicted with the estimated map, estimated trajectories (green) and odometry (purple) using a 0.5 m resolution (using the proposed extension with soft constraints). Left: warehouse aisle zig-zag dataset (traversed distance: 400 meters). Middle: warehouse loop (traversed distance: 462 meters). Right: dairy factory (traversed distance: 262 meters).

of the augmented approach is substantially more robust to the NDT grid resolution. This behavior can clearly also be seen in Fig. 9(top) which depicts the relative position error. The relative rotational error, shown in Fig. 9(bottom), is not significantly affected by the proposed approach. This is due to the fact that the odometry-based orientation estimate is typically very noisy, with a significantly larger variance than the expected rotational error of the original NDT-D2D registration approach. Analyzing these results, we note that the NDT-OM grid representation tends to smooth out details with increasing cell size. Thus, the better performance of the proposed approach for larger cell sizes indicates that compared to the original NDT-D2D algorithm, it copes better with feature-poor environments.

Finally, a comparison of the runtime complexity of scan to map registration for the different algorithms and parameter configurations is depicted in Fig. 8. We note an increase in computational complexity when using soft-constraints, which we attribute to the more complex shape of the objective function and consequently to a higher number of iterations necessary for convergence.

D. Dairy production

The "dairy production" dataset consists of more open areas and areas and with more significant features, such as larger pillars and goods (trays of milk). In general the area is more open and the sensor view is not blocked by homogeneous large structures ($e.g.$, shelves). The size of the environments makes this dataset to be less demanding, especially if the sensor cut-off distance is large and the same walls are visible from a large portion of the traversed trajectory. The prior NDT-D2D registration approach performs reasonably well, even when using large cell sizes (See Fig. 10). This can be explained by the larger and planar structures present in this environment, which are well modeled even at rough grid resolutions (large cell sizes). For larger NDT cells and lower sensor cut off distances, we note that the proposed approach results in more accurate tracking also in this environment.

E. Comparison of scan to scan registration approaches

In addition to the evaluation performed using the mapping framework the underlying scan to scan registration is evaluated in this section. Three approaches are evaluated; the original NDT-D2D algorithm, the extended version with soft constraints (NDT-D2D-SC) and a filtering approach where the odometry (with covariance) is fused with the NDT-D2D result along with a covariance estimate of the registration. The weighting of odometry and registration uses the weighted mean maximum likelihood estimate defined as:

$$\bar{\boldsymbol{p}} = \frac{\Sigma_{D2D}^{-1}\boldsymbol{p} + \Sigma^{-1}\boldsymbol{p_0}}{\Sigma_{D2D}^{-1} + \Sigma^{-1}}, \tag{6}$$

where $\boldsymbol{p}$ and $\boldsymbol{p_0}$ is the pose estimate from the NDT-D2D registration and odometry respectively and Σ_{D2D} is the estimated covariance of the registration whereas Σ is the covariance from the incremental odometry (Eq. 5). The

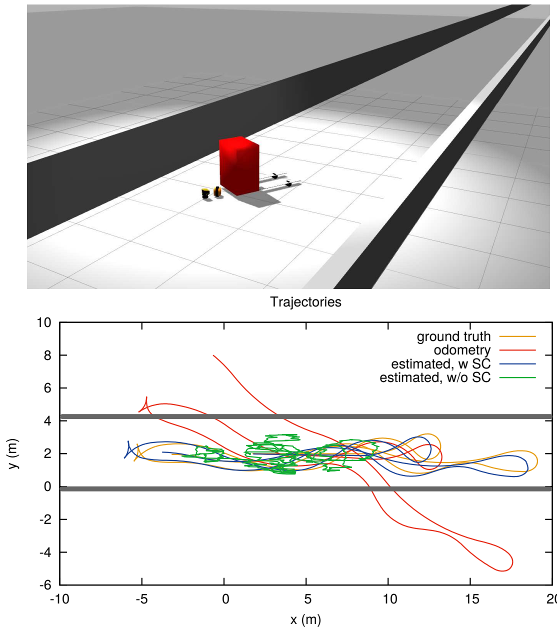

Fig. 4: Top: Endless corridor simulation environment along with the simulated steer and drive actuated forklift. Bottom: Pose estimated from from driving back and forth in the simulated endless corridor environment; w SC (with soft-constraints) and w/o SC (without soft-constraints), odometry and ground truth (obtained by querying the pose from the simulator).

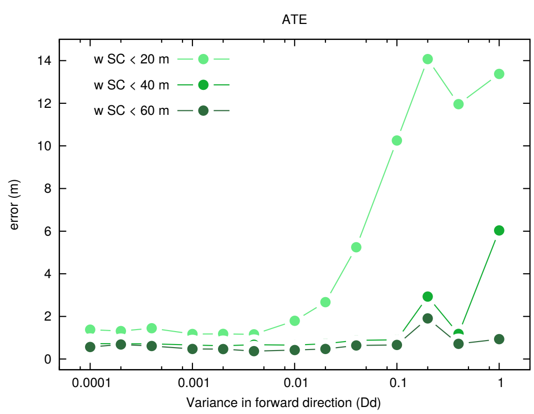

Fig. 5: "Zig-zag aisle" dataset, absolute translational error (ATE) using different D_d parameter values and a fixed NDT cell resolution of $0.8 \times 0.8 \times 0.8$ meters and with 3 different sensor cut-off distances, 20, 40 and 60 meters.

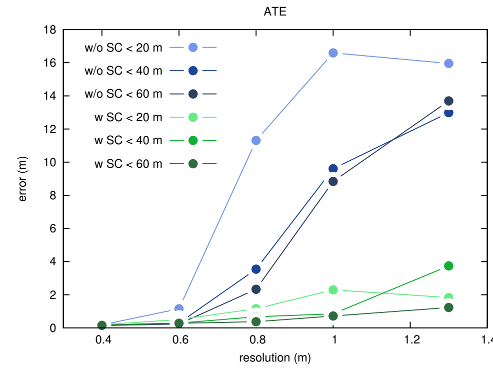

Fig. 6: "Zig-zag aisle" dataset, trajectory estimate comparisons vs. reflector based ground truth.

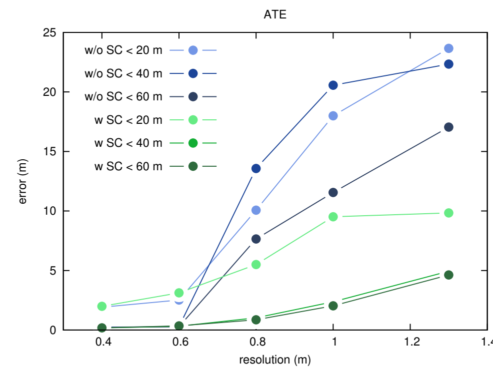

Fig. 7: "Loop" dataset, trajectory estimate comparisons vs. reflector based ground truth.

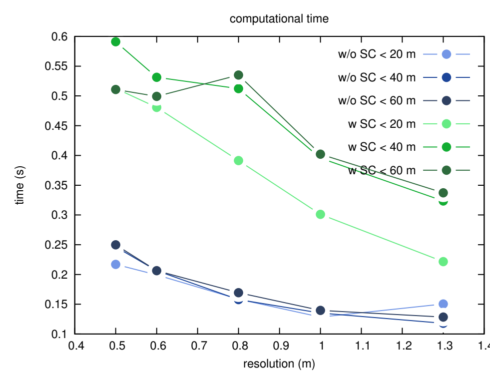

Fig. 8: "Loop" dataset, computational times for registration with and without using soft-constraints, at different resolutions.

covariance matrix Σ_{D2D} is the inverse of the Hessian matrix in the last iteration in the optimization of $f_{D2D}(\boldsymbol{p})$ using Newton's method.

To visualize the the objective functions, including the estimated relative pose estimates, a set of 2D plots were generated. In all figures, the x axis is the relative forward motion whereas the y axis contains the rotation (yaw). The origin of the plots is selected based on the objective plotted, for example, in case of the NDT-D2D objective it also use this estimate as the center of the plot. The two scans that are used is depicted in Fig. 11. In Fig. 12 the objective of the standard NDT-D2D approach is visualized along with different motion model parameters of the NDT-D2D-SC approach whereas a comparison of different resolutions is shown in Fig. 13.

Despite that these score plot only show data from a single scan pair some indications on different behavior are visible. The NDT registration is sensitive to the resolution factor where a small resolution will cause points from one laser beam ('ring') on the floor to end up in the same distribution. This would cause the floor to be represented of a set of

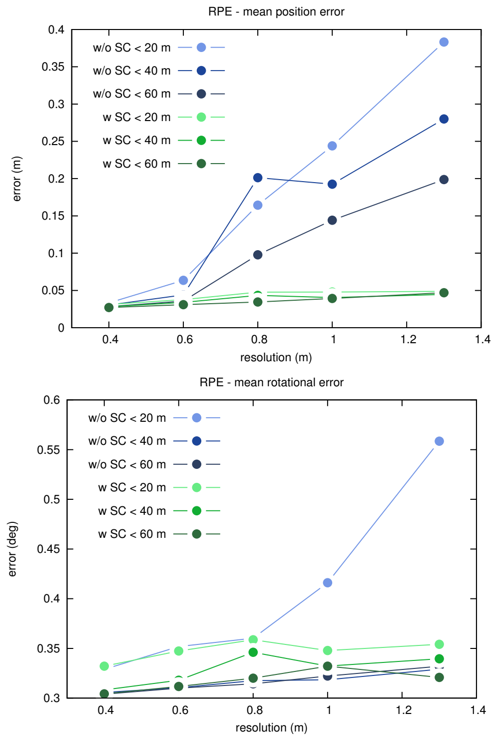

Fig. 9: "Loop" dataset, relative pose error (RPE). Top: mean position error. Bottom: mean rotational error.

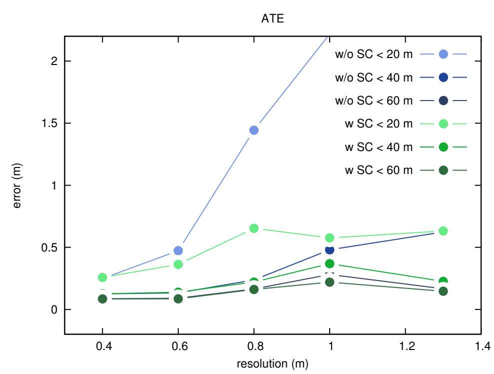

Fig. 10: "Dairy production" dataset, trajectory estimate comparisons vs. reflector based ground truth.

thin distribution along a single direction rather than flat ones (which naturally would occurs in larger resolutions), see also Table I. Note that this is one reason for the different performances between a scan to scan based approach (this section) and an approach that incrementally build local maps (previous sections) where, for example, the distributions representing the floor are more accurately modelled.

In addition to the single pair scan to scan matching the Zig-zag dataset (Fig. 6) was used where each consecutive scan pair were registered and the pose estimates are computed by incrementally adding each registration results together (a total of 3206 pairs).

The results using the default parameters $D_d = 0.004$ and $D_t = 1$ are shown in Table I where there results are far less precise compared to the results presented in Fig. 6. To

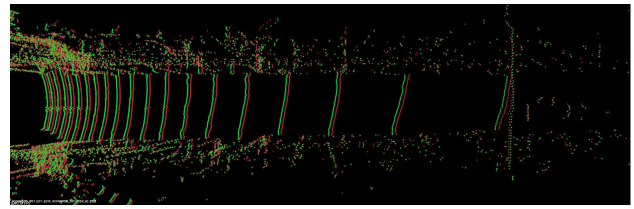

Fig. 11: Two consecutive laser scans from the warehouse data set (zig-zag) where the corresponding forward motion is along the x-axis in this image (right). The relative offset displayed in this figure are computed using the NDT-D2D with soft constraints. Contrary, the objective for the original NDT-D2D registration is smallest when the red and green laser points on the floor overlap, that is when the red points are shifted towards the left in this images indicating no forward motion, see also Fig. 12.

TABLE I: ATE registration results with resolution 0.4 and 1.0 using, $D_d = 0.004$ and $D_t = 1$.

Registration method	ATE mean (res 0.4)	ATE men (res 1.0)
NDT-D2D	15.42	6.70
NDT-D2D-SC	14.00	3.21
NDT-filter	15.15	5.78

further limit the variance on the forward motion the rotational component Dt was set to be the same as the Dd parameters in the following experiment. The results are depicted in Fig. 14 which indicates that a larger resolution in this case makes the scan to scan registration approaches less sensitive to the parameter selection. It also indicates that scan to scan registration, for the evaluated methods, is more sensitive than incrementally fusing the data into a map.

IV. Conclusions and Future Work

In this work we proposed a method to incorporate uncertainty-aware ego-motion estimates into the objective function of a registration algorithm. We evaluated our approach in an industrially-relevant AGV application scenario and noted a significant improvement in pose estimation accuracy. Our approach was shown to be especially beneficial in cases that due to the environment topology, limitations of the sensor range, or the mapping resolution, do not provide enough features to reliably estimate relative translation.

Although the motivating application of this work lies in AGVs for indoor operation, in the future we plan to extend our approach to outdoor applications, by utilizing other types of egomotion/positioning sensors, for example, IMUs. This extension will allow us to also test on popular large-scale outdoor datasets, such as the KITTI [16] or Ford Campus [17] datasets, which are currently out of the scope of our approach, as they do not provide any egomotion estimates from odometry.

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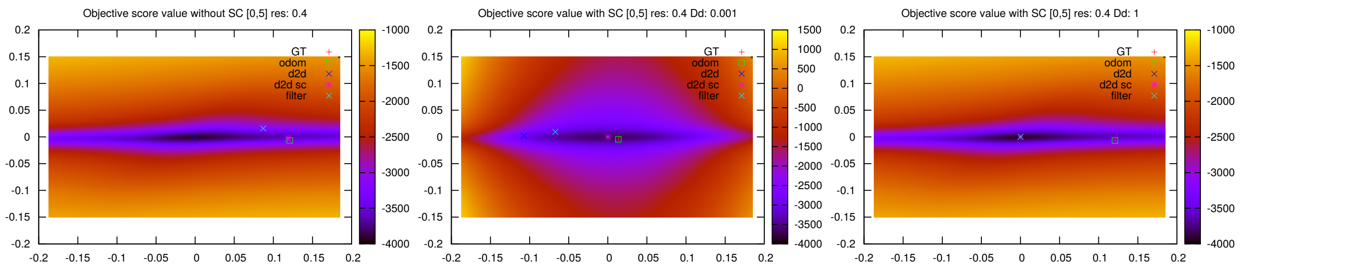

Fig. 12: Objective scores. Left: NDT-D2D where the center is plotted in the estimated pose from NDT-D2D. Middle/Right: objective scores from NDT-D2D-SC by altering the D_d motion parameter (middle $D_d = 0.001$, right $D_d = 1.$). In the latter case the influence of the soft constraint is small and all registration methods provide a similar estimate - note also the similarity in the score values between the left most figure. 'GT' - ground truth estimate from the reflector based localization system. 'odom' - odometry. 'd2d' - NDT-D2D (without soft constraints). 'd2d-sc' - NDT-D2D-SC (with soft constraints). 'filter' - the ML estimate detailed in Eq. 6.

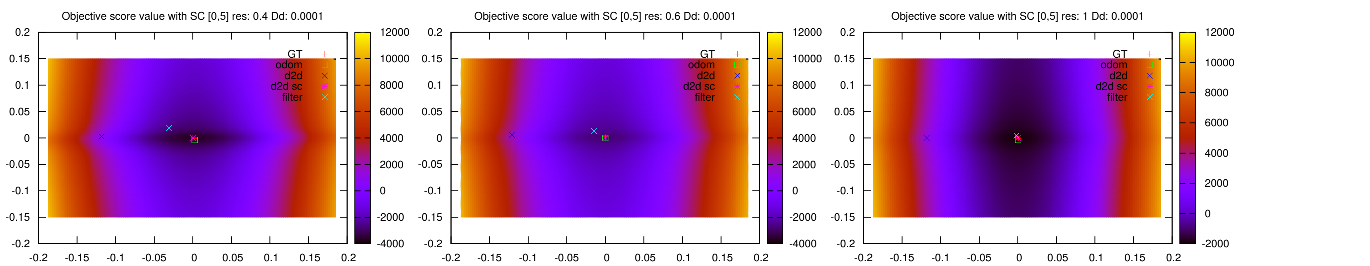

Fig. 13: Objective score with different resolutions (from left to right; 0.4, 0.6 and 1.0) from NDT-D2D-SC where the center is plotted in the estimated pose from NDT-D2D-SC registration.

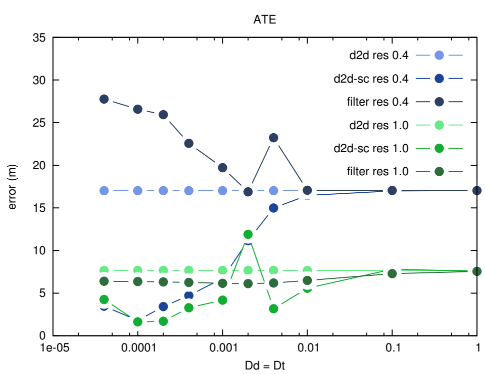

Fig. 14: Evaluation of ATE using scan to scan registration for different motion parameters $Dd = Dt$. Note that d2d (NDT-D2D) registration has no influence on the motion parameters.

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